Technical Notes

Application of Midcourse Guidance Technique to Orbit Determination

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1. Introduction

Before a manned interplanetary space flight is attempted, the midcourse guidance system of the spacecraft must be tested in an environment that resembles, as nearly as possible, the ultimate flight conditions. A near-earth orbit provides such an environment. In addition, during any interplanetary mission, the spacecraft would probably be in a near orbit of the earth and another planet for certain periods of time. In the interest of computer economy, it would be advantageous to use the same navigation and guidance procedure for all phases of the flight.

For these reasons, the application of midcourse guidance theory to the problem of guiding a spacecraft that is orbiting a planet is considered here. This note is concerned only with the navigational aspect of the guidance problem, that is, the estimation of the position and velocity of the spacecraft.

In the navigation procedure presented here, a sequence of celestial observations is assumed to be made. At each observation, the predicted value of the measured quantity is calculated from the present estimate of the position and velocity of the spacecraft and is compared to the measured value. A new estimate of position and velocity is then calculated on the basis of the comparison.

This method of navigation was first derived from Kalman's linear filtering theory.[1, 2] References 3 and 4 report on the application of Kalman's theory to spacecraft navigation. The form of the midcourse navigation procedure outlined in the next section is essentially as developed by Battin.[5]

In Sec. 3, the procedure is applied to orbital navigation, and a statistical analysis is performed. With regard to notation, both three-dimensional and six-dimensional vectors are represented by boldface symbols. A superscript T stands for the transpose of a vector or matrix. The notation $\langle \alpha \rangle$ indicates the mathematical expectation or average value of α.

2. Midcourse Navigation Theory

Let $\mathbf{r}$ and $\mathbf{v}$ be the true position and velocity, respectively, of the spacecraft. Then the six-dimensional state vector $\mathbf{x}$ is defined by

$$\mathbf{x} = \begin{pmatrix} \mathbf{r} \\ \mathbf{v} \end{pmatrix} \tag{1}$$

Similarly, the estimated state vector $\hat{\mathbf{x}}$ (the state vector calculated by the onboard computer) is given by

$$\hat{\mathbf{x}} = \begin{pmatrix} \hat{\mathbf{r}} \\ \hat{\mathbf{v}} \end{pmatrix} \tag{2}$$

It is necessary to distinguish between the estimate before incorporation of measurement data and the estimate after the incorporation. Let primed quantities indicate values before an observation and unprimed quantities values after the observation. Thus, the estimated state vector before the measurement (or the estimate after the previous time integrated to the present time) is given by

$$\hat{\mathbf{x}}' = \begin{pmatrix} \hat{\mathbf{r}}' \\ \hat{\mathbf{v}}' \end{pmatrix} \tag{3}$$

Next, define the estimation error $\mathbf{e}$ to be

$$\mathbf{e} = \hat{\mathbf{x}} - \mathbf{x} \tag{4}$$

As previously, let $\mathbf{e}'$ be the error before the measurement:

$$\mathbf{e}' = \hat{\mathbf{x}}' - \mathbf{x} \tag{5}$$

A matrix that plays a basic role in the procedure is the covariance or correlation matrix E of the estimation error $\mathbf{e}$ defined by

$$E = \langle \mathbf{e}\, \mathbf{e}^T \rangle \tag{6}$$

Also required is the extrapolated correlation matrix E' given by

$$E' = \langle \mathbf{e}'\mathbf{e}'^T \rangle \tag{7}$$

Let A be the true value of the quantity being measured, $\tilde{A}$ the measured value, and $\hat{A}$ the estimated value calculated from the estimated state vector $\hat{\mathbf{x}}'$. Define the geometry vector $\mathbf{b}$ as having components equal to the partial derivatives of the estimated quantity $\hat{A}$ with respect to the components of the estimated state vector $\hat{\mathbf{x}}'$. Then, to first order, the error in the calculated estimate of A is given by

$$\hat{A} - A = \mathbf{b}^T\mathbf{e}' \tag{8}$$

Let the error in the measurement of A be α, and let its mean squared value be $\langle \alpha^2 \rangle$. Under the assumption that all measurements are uncorrelated, the optimum linear change to be applied to the state vector is given by

$$\hat{\mathbf{x}} - \hat{\mathbf{x}}' = \mathbf{w}(\tilde{A} - \hat{A}) \tag{9}$$

The weighting vector $\mathbf{w}$ is calculated from

$$\mathbf{w} = a^{-1}E'\mathbf{b} \tag{10}$$

where the scalar a is given by

$$a = \mathbf{b}^TE'\mathbf{b} + \langle \alpha^2 \rangle \tag{11}$$

Required also is the updated correlation matrix E, which is obtained from

$$E = E' - a\,\mathbf{w}\,\mathbf{w}^T \tag{12}$$

Note that, with the navigation system in this form, any type of data can be incorporated at any time. Required only are the geometry vector $\mathbf{b}$ and the mean squared error $\langle \alpha^2 \rangle$.

After incorporation of the measurement data by means of Eqs. (9–12), the new estimated state vector $\hat{\mathbf{x}}$ is integrated ahead to the next observation time. The correlation matrix E must also be extrapolated ahead, and this may be most easily accomplished by integration of the following differential equation:

$$\dot{E}(t) = M(\hat{\mathbf{r}}, t)E(t) + E(t)M^T(\hat{\mathbf{r}}, t) \tag{13}$$

The six-dimensional matrix $M(\hat{\mathbf{r}}, t)$ is given by

$$M(\hat{\mathbf{r}}, t) = \begin{pmatrix} O & I \\ G(\hat{\mathbf{r}}, t) & O \end{pmatrix} \tag{14}$$

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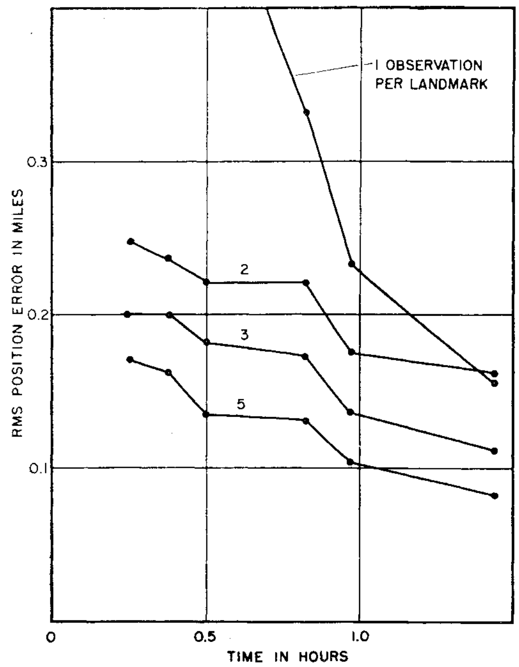

Fig. 1 Landmark uncertainty = 0.

where the matrix $G(\hat{\mathbf{r}}, t)$ is the gravity gradient matrix, and the matrices I and O are the three-dimensional identity and zero matrices, respectively.

3. Orbital Navigation

In applying the preceding theory to orbital navigation, the only observation assumed is the determination of the angles between a landmark direction and an inertial axis system fixed in the spacecraft. This observation is equivalent to the simultaneous measurement of the angles between the landmark and two stars. Hence, the measured angles are incorporated by choosing two orthogonal axes, and then using Eqs. (9–12) twice, once for each axis.

One of the major sources of error in orbital navigation using landmark sightings is the lack of knowledge of the exact positions of the landmarks, particularly if each landmark is observed several times. A method will now be derived, following Frazier's idea,[6] for determining the effect on the navigation of landmark location uncertainties.

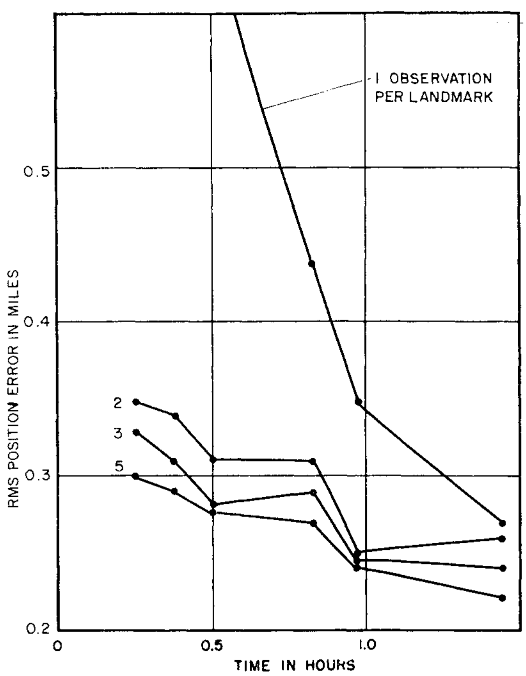

Fig. 2 Landmark uncertainty = 0.25 mile.

At this point, it should be noted from Eqs. (12) and (13) that the correlation matrix E does not depend upon either the path that the spacecraft actually follows or the measurement errors. (This is not exactly true because linearization is used with respect to the estimated trajectory rather than a predetermined reference trajectory; however, it is sufficiently true for the present purpose.) Essentially, the E matrix depends only upon the nominal trajectory, the measurement schedule, and statistical data. Thus, a statistical study of the navigation technique can be performed without resorting to Monte Carlo methods by deleting Eq. (9) from the calculations.

The correlation matrix E, however, is the matrix used on board the spacecraft, and is not a valid indication of the system performance in this situation. To determine a matrix that indicates the errors resulting from the assumption of perfectly known landmarks, let $\mathbf{e}^*$ be the true estimation error and

$$E^* = \langle \mathbf{e}^* \mathbf{e}^{*T} \rangle \tag{15}$$

be its correlation matrix.

Since the $\mathbf{b}$ vector for this type of measurement has zeros as its last three components, let

$$\mathbf{b} = \begin{pmatrix} \mathbf{h} \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{16}$$

Define $\mathbf{z}$ to be the error in the knowledge of the position of the landmark. Then, the error in the estimate of the measured quantity A, Eq. (8), must be rewritten as follows:

$$\hat{A} - A = \mathbf{b}^T \mathbf{e}^{*\prime} - \mathbf{h}^T \mathbf{z} \tag{17}$$

The error in the estimated state vector is given by Eq. (4)

$$\mathbf{e}^* = \hat{\mathbf{x}} - \mathbf{x} = (\hat{\mathbf{x}} - \hat{\mathbf{x}}') - (\mathbf{x} - \hat{\mathbf{x}}') \tag{18}$$

Use of Eqs. (5) and (9) in Eq. (18) yields

$$\mathbf{e}^* = \mathbf{w}(\tilde{A} - \hat{A}) + \mathbf{e}^{*\prime} \tag{19}$$

Then, using Eq. (17) and the fact that $\tilde{A} = A + \alpha$, Eq. (19) becomes

$$\mathbf{e}^* = (I - \mathbf{w}\,\mathbf{b}^T)\mathbf{e}^{*\prime} + \mathbf{w}\alpha + \mathbf{w}\mathbf{h}^T\mathbf{z} \tag{20}$$

It is now necessary to define the following matrices:

$$Z = \langle \mathbf{z}\mathbf{z}^T \rangle \qquad F = \langle \mathbf{e}^*\mathbf{z}^T \rangle \tag{21}$$

The correlation matrix Z of the landmark error $\mathbf{z}$ is a different constant matrix for each landmark. The updating equation for the F matrix is obtained from Eq. (20). Assuming that $\langle \mathbf{z}\alpha \rangle$ is zero, it follows that

$$F = (I - \mathbf{w}\mathbf{b}^T)F' + \mathbf{w}\mathbf{h}^T Z \tag{22}$$

The F matrix can be shown to satisfy the differential equation

$$(\dot{F}t) = M(\hat{\mathbf{r}}, t)F(t) \tag{23}$$

It is clear that the F matrix must be extrapolated ahead between all observations of the same landmark and set equal to zero when a new landmark is to be observed.

Now, using Eqs. (21) and (22), the following equation for E^* is obtained from Eq. (20):

$$E^* = (I - \mathbf{w}\mathbf{b}^T)E^{*\prime}(I - \mathbf{w}\mathbf{b}^T)^T + (\langle \alpha^2 \rangle - \mathbf{h}^T Z \mathbf{h})\mathbf{w}\mathbf{w}^T + F\mathbf{h}\mathbf{w}^T + (F\mathbf{h}\mathbf{w}^T)^T \tag{24}$$

The results of a computer simulation of the orbital navigation procedure are illustrated in the accompanying graphs.

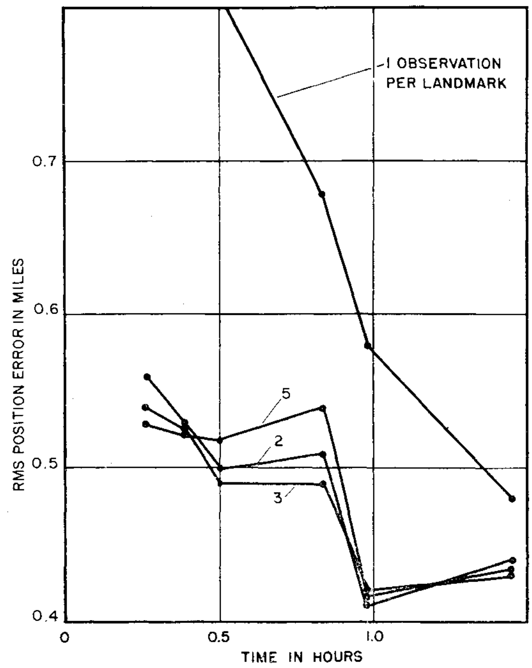

Fig. 3 Landmark uncertainty = 0.5 mile.

As a nominal trajectory, a 100-mile circular orbit around the earth from Cape Kennedy was selected. It was assumed that one landmark could be observed from one to five times at each of the following six areas of the earth: 1) West Coast of Africa, 2) Central Africa, 3) East Coast of Africa, 4) West Coast of Australia, 5) East Coast of Australia, and 6) Mexico.

Statistical data that were held constant are as follows:

1) The initial correlation matrix

$$E_0 = E_0{}^* = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{pmatrix}$$

This matrix corresponds to position and velocity injection errors of 1 mile and 10 mph in any three mutually orthogonal directions.

2) The mean squared instrument error

$$\langle \alpha^2 \rangle = 10^{-6} \text{ rad}^2$$

Figure 1 shows the results obtained with perfect landmark knowledge for various values of the number of observations per landmark. Shown are the rms position errors occurring at the conclusion of all observations of each landmark. The errors are substantially larger at intermediate times, but are omitted for convenience. Similar results are illustrated in Figs. 2 and 3 for landmark position uncertainties of 0.25 and 0.5 miles, respectively.

It is seen from these results that, the larger the landmark uncertainty, the less worthwhile it is to make additional observations of the landmark. In fact, it appears that a landmark should be observed exactly twice unless its position is known quite accurately.

The results of the preceding study indicate the effectiveness of this method of orbital navigation. An accurate estimate of the trajectory of the spacecraft can be obtained by merely a few observations of landmarks on the surface of the planet.

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Chemical and Nozzle Flow Losses in Hypersonic Ramjets

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Introduction

Because of the small differences that exist between the large values of inlet and exhaust momentum, the performance of hypersonic ramjets is extremely sensitive not only to recombination losses, but also to the exhaust nozzle divergence and frictional losses. For example, a very long nozzle will insure equilibrium flow, but large friction losses will result; on the other hand, rapidly divergent nozzles may tend to freeze the flow and minimize the friction losses, but the divergence loss will be increased. The problem reduces to finding the optimum tradeoff among all the losses. The objective of this study was to investigate the tradeoff among these losses for a supersonic combustion ramjet flying at a Mach number of 15 and an altitude of 150,000 ft.

Analysis

For the supersonic combustion ramjet, the nozzle entrance conditions are dependent upon the inlet performance and mode of combustion. The following assumptions were made concerning the processes: 1) the value of inlet kinetic energy efficiency η_k (referred to stagnation conditions) is constant at 0.990; 2) the inlet diffuser velocity ratio V_2/V_1 is a variable parameter; 3) hydrogen and air at stoichiometric proportions are burned in a constant area duct with no loss in combustion efficiency; 4) the effective fuel velocity for performance calculations is 6400 fps including losses due to friction and blockage drag; 5) the inlet capture area A_1 is 100 ft^2 and the nozzle exit area A_4 is 150 ft^2; 6) the flow entering the nozzle is in equilibrium and one-dimensional; 7) the flow through the nozzle is one-dimensional; and 8) axisymmetric conical nozzles are used for calculating all losses.

Analytical procedures for calculating the extent of recombination of complex chemical reactions in adiabatic one-dimensional flow systems have been described in published form elsewhere.[1–4] The equations and computational procedure,

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